The Modified Series Model for an Abrupt-Junction Varactor Frequency Doubler

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Abstract—The theoretical performance of an abrupt-junction high-quality epitaxial varactor diode used as a harmonic frequency doubler is considered. The modified series model for the device, consisting of a nonlinear elastance (reciprocal of capacitance) and a nonlinear resistance, is presented. A comparison is made between the performance limits derived for the modified series model and similar results based on the conventional series model. In addition to dissipating some power, the charge-variable series resistance contributes to the frequency conversion of the device.

I. Introduction

THE THEORETICAL performance of an abrupt-junction high-quality epitaxial varactor diode used as a harmonic frequency doubler will be considered in this paper. We will determine the loading conditions that yield optimum conversion efficiency. In most previous analyses of varactor harmonic doublers, the varactor has been represented by the series model proposed originally by Uhlir [2]. This model consists of a constant series resistance (which represents the loss in the neutral regions of the diode, the leads, and the contacts), and a nonlinear (charge or voltage dependent) capacitance (which represents the "varying" charge storage properties of the space-charge layer on both sides of the metallurgical junction).

It has been found [3], [4] that the series resistance in high-quality epitaxial varactors is no longer constant. Keywell has shown that the resistance at the diode's breakdown voltage may be reduced to 1/3 of its zero bias value. With larger variations predicted in future devices, a modified series model for the varactor must be incorporated in any accurate analysis. Engelbrecht [5] has examined these effects on the operation of varactor frequency converters. Pucel [6] has considered the conditions for parametric amplification with nonlinear capacitance-resistance elements.

II. The Modified Series Model

Epitaxial varactors are made by the diffusion of impurities into a high-resistivity layer, the epitaxial layer, on a low-resistivity substrate waver. The variable capacitance is caused by the variation of the width of the space-charge layer near the junction [1], [7]. Thus, the thickness of the epitaxial base layer varies with applied bias. This variation in the epitaxial base material is of sufficient magnitude to cause a large variation in series resistance, when varying voltage is applied to the device.

In many epitaxial devices, it can be assumed that the space-charge layer has swept out the entire epitaxial layer at the breakdown voltage.[1] The capacitance is at a minimum (or the elastance, the reciprocal of capacitance, is at a maximum) at the breakdown voltage, and the contribution to the series resistance from the epitaxial layer is zero, under this condition.

Consider an asymetrically doped, one dimensional, abrupt-junction, epitaxial varactor. D represents the space charge layer width; A, the transverse area; ϵ, the permittivity of the lightly doped side; and ρ, the resistivity of the lightly doped side. Then the varying elastance is [1]

$$S = \frac{D}{\epsilon A} \tag{1}$$

and the varying portion of the series resistance is [1]

$$R_s = \frac{\rho}{A}(D_{\max} - D). \tag{2}$$

$D_{\max}$ is the maximum width of the space-charge layer, which we will assume includes the entire epitaxial layer.

These two equations describe the modified series model illustrated in Fig. 1. It consists of a nonlinear elastance in series with a nonlinear resistance. The charge, q, stored in either half of the space-charge layer, is directly proportional to the space-charge layer width D (for an abrupt junction device). Thus, the elastance and varying portion of the series resistance

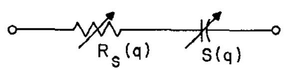

Fig. 1. The modified series model for a high-quality epitaxial varactor diode.

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[1] In epitaxial units, this is of the order of 100 volts.

can also be given by

$$S = \frac{S_{\max}}{Q_B} q \tag{1a}$$

and

$$R_s = R_{\max}\left(1 - \frac{q}{Q_B}\right). \tag{2a}$$

Here, Q_B is the charge at breakdown voltage,

$$S_{\max} = \frac{D_{\max}}{\epsilon A} \tag{3}$$

and

$$R_{\max} = \frac{\rho D_{\max}}{A}. \tag{4}$$

In this discussion, we will be concerned with a nominally driven diode (the diode charge will vary from zero to Q_B). We will assume that the constant resistance of the leads and contacts of the diode can be neglected. Thus, (2a) represents the total series resistance, seen to vary between zero and $R_{\max}$ under nominal drive.

III. Harmonic Doubler Performance Equations

A set of equations are derived that will be used to yield the optimum conversion efficiency conditions for a varactor harmonic doubler. In this analysis, we assume that the varactor is coupled to a source at frequency ω and that it delivers power to a load $Z_2 = R_2 + jX_2$, at frequency 2ω. We also assume that the coupling from source to varactor and from varactor to load is through lossless band-pass filters. The input filter only passes a small band at the fundamental, while the output filter only passes a small band at the second harmonic. The varactor will be represented by the modified series model of Fig. 1 and (1) and (2).

For later use, we introduce the concept of the diode's cutoff frequency. This factor has been defined in the past [1] as

$$\omega_c = \frac{S_{\max}}{R_s} \tag{5}$$

where R_s refers to the "constant" series resistance of the diode. In our analysis, the resistance varies, and the above definition is inadequate. The author feels the average[2] series resistance should be used in the cutoff-frequency expression. A true average, under ac excitation, will depend on the shape of the resistance vs. time waveform. However, for simplicity, the cutoff frequency will be defined in terms of a fixed resistance. Thus, the use of a series cutoff (sc) resistance R_{sc} will be defined as

$$R_{sc} = \frac{R_{\max}}{2}. \tag{6}$$

Using this for R_s in (5), we have

$$\omega_c = \frac{2S_{\max}}{R_{\max}} = \frac{2}{\rho\epsilon} = 2\omega_r \tag{7}$$

where ω_r is the dielectric relaxation frequency of the diode. From (7), we see that the cutoff frequency and the dielectric relaxation frequency of varactors are the same order of magnitude. Remember that the relaxation time (reciprocal of the relaxation frequency) is the time required for a charge density disturbance to decrease to 37 percent of its initial value. Thus, when the input frequency approaches the cutoff frequency (or correspondingly, the relaxation frequency), we are led to suspect that the assumption of an abrupt charge distribution, which led to the modified series model, is violated. Perhaps more appropriate high frequency models are in order (see Section VI).

Returning to Fig. 1, we can state the large signal equations of motion of the modified series model as

$$V(t) = R_s(t)i(t) + \int S(t)i(t)dt. \tag{8}$$

Writing each of the time varying quantities in a Fourier series, with the input frequency as the fundamental, we are able to transform the foregoing nonlinear differential equation into a set of coupled algebraic equations. In most instances, these are easier to solve than the nonlinear differential equation, since the loading constraints are usually expressed in the frequency domain. Following the above procedure, it is possible to show that V_1 and V_2 (the Fourier voltage coefficients at ω and 2ω, respectively) must satisfy (8) and (10)[3]

$$V_1 = \left[\frac{S_{\max} - S_0}{\omega_r} + \frac{S_0}{j\omega}\right] I_1 + \left[\frac{1}{j\omega} - \frac{1}{\omega_r}\right]\frac{S_1^* I_2}{2} \tag{9}$$

$$V_2 = \left[\frac{1}{j2\omega} - \frac{1}{\omega_r}\right] S_1 I_1 + \left[\frac{S_{\max} - S_0}{\omega_r} + \frac{S_0}{j2\omega}\right] I_2. \tag{10}$$

With a load constraint of

$$V_2 = -Z_2 I_2 \tag{11}$$

and an input condition of

$$V_1 = Z_{\text{in}} I_1 \tag{12}$$

and using (9) and (10), equations can be derived for the various diode parameters. We define the normalization

[2] Averaged over one period of the input frequency.

[3] Where S_1^* is the complex conjugate of S_1.

power P_{norm} as

$$P_{\text{norm}} = \frac{(V_B + \phi)^2}{R_{sc}} \quad (13)$$

where V_B is the diode's breakdown voltage and ϕ is the contact potential. Then, the various diode quantities are [8]

$$R_2 = R_{\max}\left[\frac{(\omega_r^2 + 4\omega^2)^{1/2} m_1^2 \cos\theta}{4\omega m_2} - (1 - m_0)\right] \quad (14)$$

$$X_2 = R_{\max}\left[\frac{(\omega_r^2 + 4\omega^2)^{1/2} m_1^2 \sin\theta}{4\omega m_2} + \frac{m_0 \omega_c}{4\omega}\right] \quad (15)$$

$$R_{\text{in}} = R_{\max}\left\{(1 - m_0) - \frac{m_2 \omega_r}{(\omega_r^2 + 4\omega^2)^{1/2}} \cdot \left[\frac{(2\omega^2 - \omega_r^2)}{\omega\omega_r}\cos\theta + 3\sin\theta\right]\right\} \quad (16)$$

$$P_{\text{in}} = 16\left(\frac{\omega}{\omega_c}\right)^2 m_1^2 \left(\frac{R_{\text{in}}}{R_{\max}}\right) P_{\text{norm}} \quad (17)$$

and

$$P_{\text{out}} = 64\left(\frac{\omega}{\omega_c}\right)^2 m_2^2 \left(\frac{R_2}{R_{\max}}\right) P_{\text{norm}}. \quad (18)$$

In the above equations, θ is the angle of the impedance[4]

$$R_{\max}(1 - m_0) + \frac{S_0}{j2\omega} + Z_2,$$

the m_k's are the normalized elastance coefficients of Penfield and Rafuse[5] given by

$$m_k = \frac{|S_k|}{S_{\max}} \quad (19)$$

and the S_k's are the elastance coefficients at frequency $k\omega$. R_{in} is the real part of the input impedance, P_{in} is the average power delivered to the varactor, and P_{out} is the average power delivered to the load.

The conversion efficiency η, a measure of the ability of the varactor to convert fundamental to second harmonic power, is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}. \quad (20)$$

Using the foregoing equations, with the normalized frequency

$$\bar{\omega} = \frac{2\omega}{\omega_r} = \frac{4\omega}{\omega_c} \quad (21)$$

we have, for the conversion efficiency,

$$\eta = \frac{\dfrac{4m_2^2}{m_1^2}\left[\dfrac{(1 + \bar{\omega}^2)^{1/2} m_1^2 \cos\theta}{2\bar{\omega} m_2} - (1 - m_0)\right]}{(1 - m_0) - \dfrac{m_2}{(1 + \bar{\omega}^2)^{1/2}}\left[\dfrac{\bar{\omega}^2 - 2}{\bar{\omega}}\cos\theta + 3\sin\theta\right]}. \quad (22)$$

For a given input frequency and output phase angle θ the efficiency is only a function of m_0, m_1, and m_2. A breakdown relation between m_1 and m_2 will assist in determining the optimum value of conversion efficiency.

IV. Breakdown Relation

The Fourier series for the diode charge is

$$q(t) = q_0 + 2q_1 \sin\omega t + 2q_2 \sin(2\omega t + \alpha). \quad (23)$$

The angle α is the phase angle between the fundamental and second harmonic currents; it is given by

$$\alpha = -\tan^{-1}\bar{\omega} - \theta. \quad (24)$$

Using (23) in (1a) we have

$$S(t) = S_0 + 2S_1 \sin\omega t + 2S_2 \sin(2\omega t + \alpha). \quad (25)$$

Normalizing this in terms of $S_{\max}$

$$m(t) = \frac{S(t)}{S_{\max}} = m_0 + 2m_1 \sin\omega t + 2m_2 \sin(2\omega t + \alpha). \quad (26)$$

We have stated that $S(t)$ will vary from 0 to $S_{\max}$ under nominal drive. Thus, under the same condition, the normalized elastance $m(t)$ will vary from zero to unity. By imposing this condition on $m(t)$ in (26), we obtain the limiting relation for m_1 and m_2. This limiting relation, referred to by Penfield and Rafuse [1] as the "breakdown limit," divides the m_1, m_2 plane into two regions. For values of m_1 and m_2, below and on the breakdown curve, doubler action is realizable, while the region above is not achievable.

At a given input frequency, the point of maximum efficiency will lie on the breakdown curve [1]. Thus, a knowledge of the breakdown limit will be of assistance in determining the conditions for optimum conversion efficiency. The relation given by Penfield and Rafuse [1] is used for the case of in-phase elastance coefficients. In our analysis, however, these coefficients will generally be out of phase. The dephasing complicates the calculations and necessitates the use of a digital computer.

The breakdown curve was obtained by finding the pairs of m_1, m_2 that cause $m(t)$ in (26) to vary from zero to unity during one cycle of the input frequency. The average normalized elastance m_0 can be determined from the knowledge of the breakdown values of m_1 and m_2 and the point where $m(t)$ attains a maximum or minimum.

V. Results

After the breakdown relation was derived, optimum conversion efficiency was considered. We initially optimized the efficiency on a single breakdown curve with

[4] This is the impedance of the output loop with the varactor replaced by its average impedance.

[5] Penfield and Rafuse [1], p. 86.

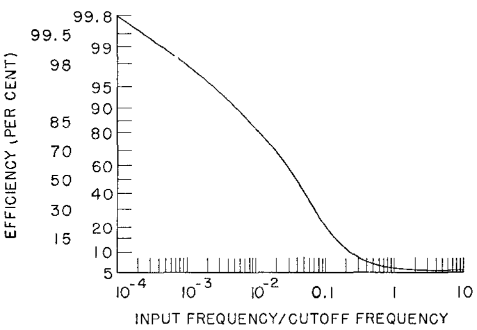

Fig. 2. Maximum efficiency based on the modified series model.

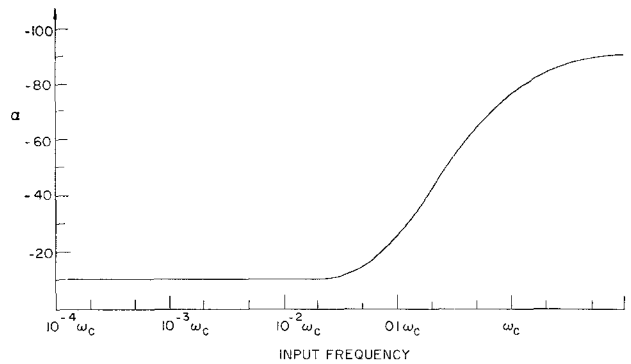

Fig. 3. Value of alpha that yields the maximum efficiency of Fig. 2.

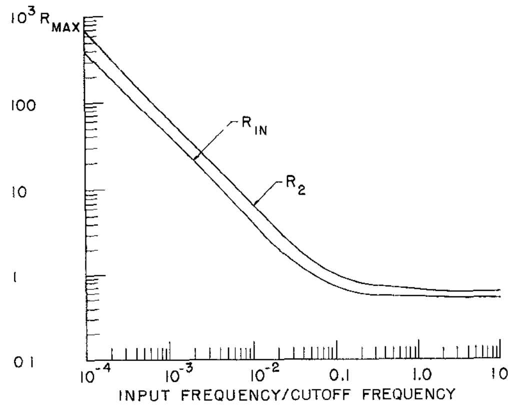

Fig. 4. Input and load resistance at point of maximum efficiency.

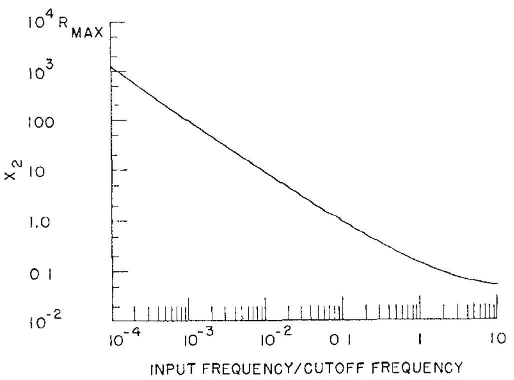

Fig. 5. Load reactance at point of maximum efficiency.

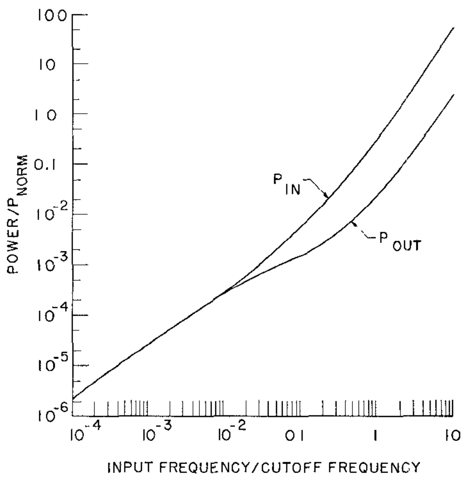

Fig. 6. Input and output power at point of maximum efficiency.

α held constant. This data was examined to yield the true optimum efficiency at a given input frequency. Figure 2 shows this optimum efficiency and Fig. 3 gives the value of α necessary to achieve it. In the latter figure, the optimum value of α is $-10°$ at low frequencies, whereas it approaches $-90°$ at high frequencies. The input and load resistance, the load reactance, and the input and output power necessary to achieve the maximum efficiency of Fig. 2 are given in Figs. 4 to 6.

The results for $\alpha = -90°$ are unusual and deserve mention. It was found that the conversion efficiency was independent of input frequency under this condition. From (22), the efficiency at $\alpha = -90°$ is

$$\eta = \frac{\dfrac{4m_2^2}{m_1^2}\left[\dfrac{m_1^2}{2m_2} - (1 - m_0)\right]}{(1 - m_0) - m_2}. \tag{27}$$

Let us consider a possible explanation for the frequency independence of the conversion efficiency at $\alpha = -90°$. Under this condition, the output current lags the input current by 90°. In this case, the varying elastance will not contribute to the real part of the input impedance. Thus, no real power conversion will take place in the varying elastance. In other words, all the frequency conversion is taking part in the nonlinear series resistance for $\alpha = -90°$. The resistive impedance is independent of frequency.[6] Thus, its contribution to the conversion efficiency will not depend on the drive frequency.[7]

Table I presents the optimum efficiency based on Uhlir's model, in addition to that based on the modified series model. From this, we see that both models yield the same results at low frequencies. For frequencies above $0.1\omega_c$, the results based on the modified series

[6] Neglecting skin effect.

[7] It is possible to show that a hypothetical charge variable resistor, used as a frequency doubler, will provide a conversion efficiency independent of frequency [8].

TABLE I

COMPARISON OF MAXIMUM EFFICIENCY OF AN ABRUPT-JUNCTION VARACTOR DOUBLER BASED ON THE CONVENTIONAL AND MODIFIED SERIES MODELS

ω/ω_c	η'_{max} (based on the modified series model)	η_{max} (based on the conventional series model)*
10^{-4}	0.9980	0.9980
2×10^{-4}	0.9960	0.9960
4	0.9921	0.9921
8	0.9843	0.9842
10^{-3}	0.9805	0.9804
2×10^{-3}	0.9614	0.9612
4	0.9248	0.9243
8	0.8568	0.8558
10^{-2}	0.8252	0.8239
2×10^{-2}	0.6885	0.6853
4	0.4943	0.4873
8	0.2886	0.2707
0.1	0.2332	0.2100
0.2	0.1154	0.0776
0.4	0.0712	0.0226
0.8	0.0577	0.0060
1	0.0562	0.0039
2	0.0541	0.00097
4	0.0535	0.00024
8	0.0535	0.00006
10	0.0535	0.00004

* P. Penfield, Jr., and R. P. Rafuse, *Varactor Applications*. Cambridge, Mass.: M.I.T. Press, 1962, Fig. 8.6, p. 329.

model give higher conversion efficiencies. This is due to the frequency conversion occurring in the charge variable resistance.

VI. CONCLUSIONS

We have shown that the charge-variable series resistance of a high-quality epitaxial varactor contributes to the frequency conversion, as well as to the diode losses. This effect is important at high frequencies, where the conversion in the varying elastance is being reduced. Since we are dealing with an ideal model of the device, our results are fundamental performance limits. There is a need for experimental verification of the results presented here. With the continuing acceleration of device technology, it should be possible to conduct the necessary experiments in the not too distant future.

It was stated above that the static charge distribution of an abrupt-junction diode is questionable at frequencies near the diode's dielectric relaxation frequency. The author is considering more appropriate high-frequency models. In one model under consideration, allowances are made for the flow of displacement current (including the usual conduction current) in the neutral regions. In another model considered, the restriction of a distinct boundary between the depletion and neutral regions is removed. The only assumptions made in connection with this model are that the diode is asymetrically doped, one end of the lightly doped side is fully depleted of mobile carriers, and the other end (at the contact) is neutral. Preliminary calculations indicate that these two models yield results similar to those of the modified series model presented here. When work on the high-frequency models is completed, the results will be presented in the literature.

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